Decoupling Control of a Distillation Column

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Luyben (*1*) recently discussed the use of linear decoupling control for a binary distillation column. An ideal decoupling and a simplified decoupling approach were presented. A signal flow graph analysis is presented here to show how these two methods relate to previous decoupling procedures and to suggest other possible control approaches for this problem.

The specified control problem was the regulation of both the overhead composition x_D and bottoms composition x_B at their respective setpoints by the manipulation of the reflux rate R and the vapor boilup rate V_B in the face of such unmeasured disturbances as the feed composition x_F and feed flow rate F_L. Luyben reduced the Laplace-transformed system differential equations (*2*) to the linear transfer matrix model commonly known as the *P*-structure. It is convenient here to also define the variables as perturbations from the desired steady state. This leads to the *P*-structure equations used by Luyben:

$$\begin{bmatrix} \hat{x}_D \\ \hat{x}_B \end{bmatrix} = \left[\begin{array}{cc:cc} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \end{array}\right] \left\{\begin{array}{c} \hat{x}_F \\ \hat{F}_L \\ \hdashline \hat{R} \\ \hat{V}_B \end{array}\right\} \tag{1}$$

which can be written as

$$[Y^C] = [P^{C,U} \ \vdots \ P^{C,M}] \begin{bmatrix} X^U \\ \hdashline X^M \end{bmatrix} \tag{2}$$

Equation (1) can be represented graphically by the signal flow graph of Figure 1. Various decoupling approaches have been suggested for such multivariable processes.

FEEDBACK DECOUPLING OF THE *P*-STRUCTURE

This early approach was developed by Kavanagh (*3*), Mesarovic (*4*), Bollinger and Lamb (*5*), and others. A feedback controller matrix H, shown in Figure 2 as the controllers h_{11}, h_{12}, h_{21}, and h_{22} provides decoupling and primary feedback control. Decoupling design consists of selecting an H matrix which will diagonalize the controlled process matrix $P' = \{[I + P^{C,M} H]^{-1} P^{C,M}\}$. No general procedure exists for selecting the most effective set of feedback controllers from the infinite possible set. This is because the decoupling and primary feedback control roles are contained in all of the elements of H and cannot be separated. Most reported procedures first involve a specification of the desired dynamics of the controlled process (specification of P') and then a search for the H matrix that will satisfy this specification. Since H is contained in P', this can lead to extensive effort and realizability problems.

The ideal decoupling presented by Luyben (*1*) corresponds to this feedback decoupling of the *P*-structure. This can be seen in the ideal decoupling control illustrated in Figure 3. Since the uncoupling and primary feedback control roles cannot be separated in this approach, the controllers B_1 and B_2 can be incorporated into the D controllers to give a control structure identical to Figure 2. The

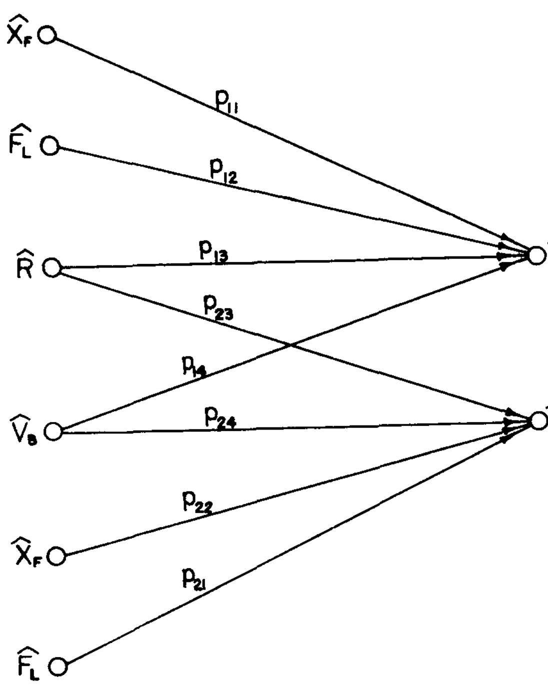

Fig. 1. *P*-structure for distillation column example.

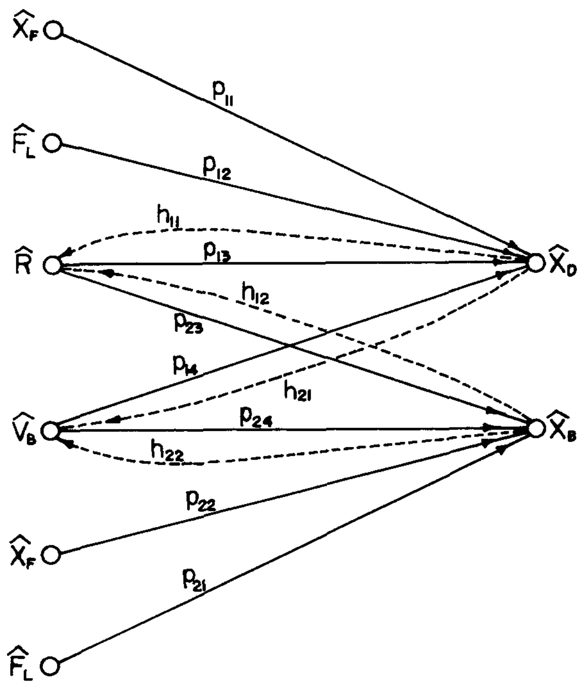

Fig. 2. Feedback decoupling control of *P*-structure.

intuitive diagonalization used by Luyben is an interesting way to eliminate the trial-and-error selection of a P' matrix. However, there is no assurance that this is an effective diagonalization choice. Other choices would result in decoupled subsystems with different dynamic response behavior.

FEEDFORWARD DECOUPLING OF THE *P*-STRUCTURE

Mesarovic (*4*) suggested that the *P*-structure could be decoupled using feedforward decoupling control rather than feedback decoupling control. It was noted that this approach led to difficulties similar to feedback decoupling. As an illustration of the problems involved, consider the *P*-structure shown in Figure 4 where the feedforward decoupling controller matrix consists of the two elements f_{f1} and f_{f2}. If the path $f_{f1}p_{13}$ is exactly the negative of the path p_{14}, it would seem that $\hat{V}_B$ cannot affect $\hat{x}_D$. Letting $f_{f1}p_{13} = -p_{14}$ gives

$$f_{f1} = -(p_{14}/p_{13}) \tag{3}$$

Likewise

$$f_{f2} = -(p_{23}/p_{24}) \tag{4}$$

It would seem that decoupling has been achieved with two controllers whose design is straightforward. Further, the primary feedback control could then be independently designed using single-variable feedback techniques. However, the two feedforward controllers do not generally decouple the process. Changes in one controlled variable due to disturbances can produce changes in another controlled variable. This feedback intercoupling, which usually exists, does not appear explicitly in the *P*-structure. However, such intercoupling is evident in the differential equation structure (*6*).

This implicit intercoupling between controlled variables, which is implicitly treated in feedback decoupling, makes the feedforward decoupling of the *P*-structure a fairly complex problem. In general, additional controllers would probably be needed and the difficulties would correspond to those of feedback decoupling.

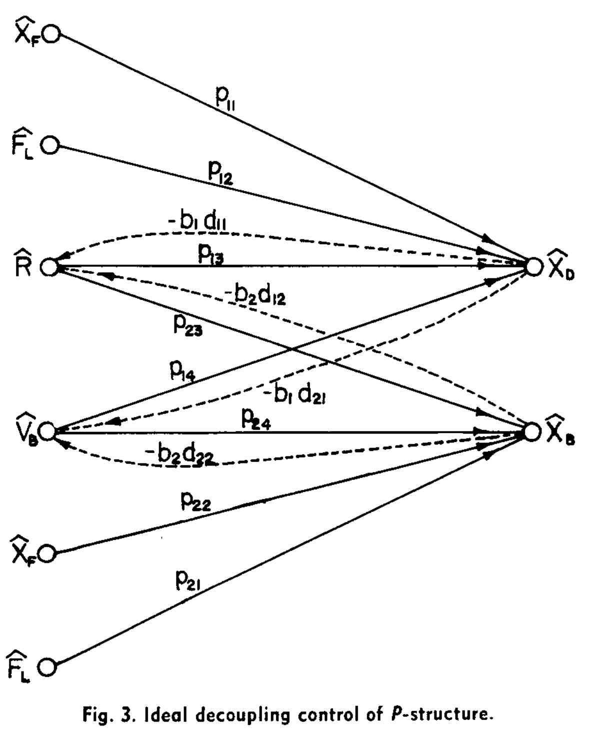

Fig. 3. Ideal decoupling control of *P*-structure.

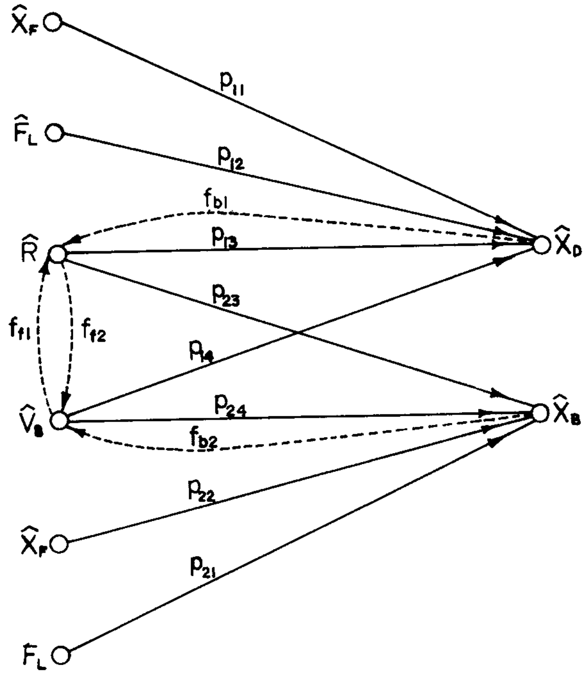

Fig. 4. Feedforward decoupling attempt for *P*-structure.

The simplified decoupling of Luyben (*1*) uses the above controller design approach. The controllers are actually used with a feedback-decoupled *P*-structure in an approximate manner. The resulting structure is similar to Figure 2, except that the controllers $-b_2d_{11}$ and $-b_1d_{11}$ now are $-b_2$ and $-b_1$.

FEEDBACK OF *V*-STRUCTURE

In order to overcome the limitations of the above approaches, Mesarovic (*4*) first introduced the fundamental concept that there are an infinite number of internal process structures corresponding to any *input-output* behavior and that each choice is an equally valid representation of that behavior. He used this concept to show that a terminally-equivalent structure with feedback intercoupling, called the *V*-structure, could be obtained from the *P*-structure. He also showed that this could be feedback-decoupled in a straightforward method that was independent of the primary feedback control. Greenfield and Ward (*7*) extended this approach to include the common situation where the number of process inputs was greater than the number of process outputs.

Consider the *V*-structure of the same distillation example, which is shown by the solid lines of Figure 5. The functions can be written in terms of the *P*-structure functions as

$$F^D = \begin{bmatrix} f_{11} \\ f_{22} \end{bmatrix} = [\{[P^{C,M}]^{-1}\}^D]^{-1} \tag{5}$$

$$V^+ = \begin{bmatrix} 0 & v_{12} \\ v_{21} & 0 \end{bmatrix} = -\{[P^{C,M}]^{-1}\}^+ \tag{6}$$

$$D^{C,U} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = [P^{C,M}]^{-1}\ P^{C,U} \tag{7}$$

Alternately, these functions could be obtained directly in terms of the original differential equation parameters.

The feedback decoupling controllers, shown as the dashed lines f_{c1} and f_{c2} can be designed using the invari-

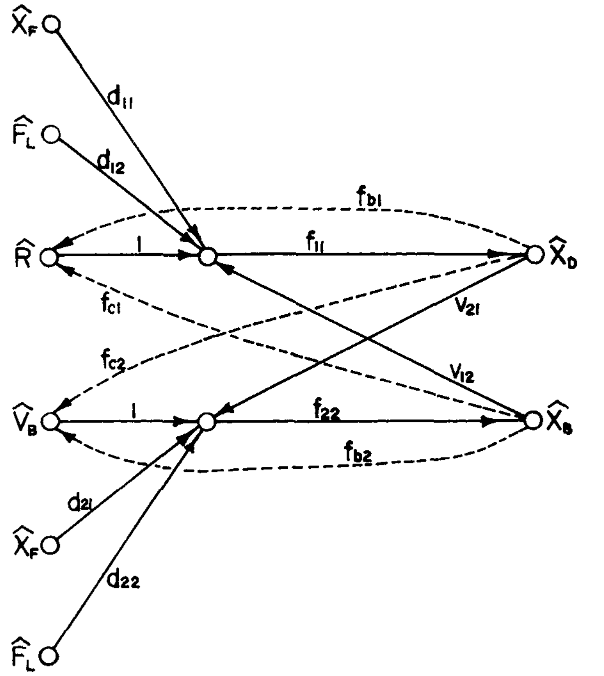

Fig. 5. Feedback decoupling control of V-structure.

ance principle. Referring to Figure 5, the condition $v_{12} = -(f_{c1})(1)$ gives

$$f_{c1} = -v_{12} \tag{8}$$

and the condition $v_{21} = -(f_{c2})(1)$ results in

$$f_{c2} = -v_{21} \tag{9}$$

A matrix generalization of this approach is available (7) for more complex cases.

The feedback decoupling controllers f_{c1} and f_{c2} decouple the linearized process. The primary feedback controllers f_{b1} and f_{b2} can then be designed using single variable feedback control methods. This V-structure approach separates the design of decoupling and primary feedback control into independent direct steps that can be carried out serially. Other advantages and limitations are noted elsewhere (7). Incidentally, feedforward compensation of $\hat{x}_F$ and $\hat{F}_L$, if measurable, could also be included. The design of these feedforward compensators is a direct application (7) of the invariance principle.

ADDITIONAL REMARKS

If only input-output data are available to develop a process model, the feedback decoupling of the V-structure model appears to be a good decoupling control approach. For this distillation example, however, the differential equation model developed earlier by Luyben (2) offers a process model of higher informational content from a control viewpoint. The differential equation model exhibits both feedforward and feedback intercouplings. The structural analysis method (6) uses this additional information to design decoupling controllers with a straightforward application of the invariance principle. In addition, feedforward compensation of measurable disturbances and control actuated by internal state variables are independent features that can be treated by a structural analysis. Another multivariable regulator control method that could be used with the differential equations of this distillation process is the modal analysis method (8 to 10).

In all of the above, it has been assumed that realizable, practical approximations to the controller functions can be obtained by frequency domain techniques, such as those used by Luyben (1, 2). The effects of nonidealities such as constraints, model error, measurement error, nonlinearities, and noise have been neglected. Further, it should be pointed out that any decoupling introduces an additional performance criterion (decoupling)and, in this sense, is probably suboptimal. Yet, decoupling does seem to achieve "near-optimal" control (11) while relating closely to conventional process control practice.

ACKNOWLEDGMENT

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NOTATION

b	=	feedback controller
d	=	ideal decoupling controller, V-structure element
D	=	V-structure matrix
f	=	V-structure element
F^D	=	V-structure matrix
f_b	=	primary feedback controller
f_c	=	feedback decoupling controller
f_f	=	feedforward decoupling controller
h	=	feedback controller element
H	=	feedback controller matrix
p	=	process transfer function
$P^{C,U}$	=	partitioned process transfer matrix
$P^{C,M}$	=	partitioned process transfer matrix
P'	=	controlled system transfer matrix
R	=	Laplace-transformed reflux variable
v	=	V-structure element
V^+	=	V-structure matrix
V_B	=	Laplace transformed vapor boiling variable
x_B	=	Laplace-transformed bottoms composition variable
x_D	=	Laplace-transformed distillate composition variable
x_F	=	Laplace-transformed feed composition variable
X^M	=	manipulative input vector
X^U	=	unmeasured input vector
Y^C	=	controlled variable vector
$\wedge$		indicates perturbation of variable from steady state

Superscripts

D	=	diagonal of the indicated matrix
$+$	=	indicates off-diagonal elements of indicated matrix

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